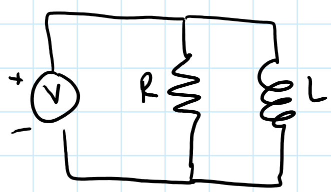
Recitation - 2/14/2025

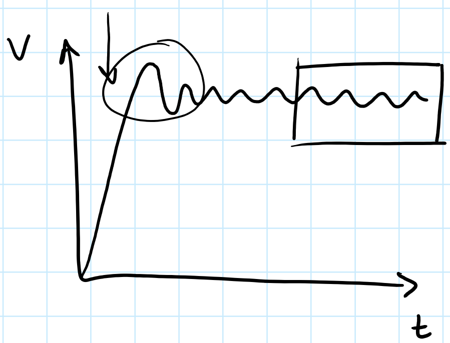
Friday, February 14, 2025 12:41 PM

1. Phasors



→ Solve for $v(t)$ → Time Domain (Transient + Steady-state)
→ Phasor Domain (Steady-state)

Phasors → Amplitude, phase



Example for phasors:

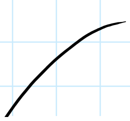
$$v_s(t) = \overset{\text{Amplitude}}{A}\cos(\underbrace{\omega t}_{\text{frequency}} + \underbrace{\phi}_{\text{phase}}) \Rightarrow v_s(t) = Re\left\{ A e^{j(\omega t + \phi)} \right\}$$

$$v_s(t) = Re\left\{ A \underbrace{e^{j(\omega t+\phi)}} \right\} = Re\left\{ \underbrace{A e^{j\phi}}_{\tilde{V}} e^{j\omega t} \right\}$$

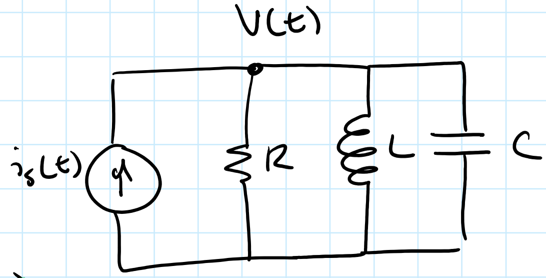
$$\tilde{V}_s \equiv Ae^{j\phi}$$

Amplitude, phase, @ $t = t$

$$\underbrace{v_s(t) = Re\left\{ \tilde{V}_s e^{j\omega t} \right\}}_{\text{time domain version.}}$$



time domain version.



$$\Rightarrow \quad i_s(t) = \frac{v(t)}{R} + \underbrace{i_L(t)}_{} + C\frac{dv(t)}{dt}$$

$$i_s(t) = \frac{v(t)}{R} + \frac{1}{L}\int v(t)\,dt + C\frac{dv(t)}{dt}$$

$$\frac{d}{dt}(\tilde{V}_s) = (j\omega)\tilde{V}_s$$

$$v_L(t) = L\frac{di_L(t)}{dt}$$

$$\int \hat{V}_s dt = \frac{1}{j\omega}\tilde{V}$$

$$\frac{1}{L}\int v_L(t)\,dt$$

$$\hat{I}_s = \frac{\tilde{V}}{R} + \frac{1}{j\omega L}\cdot\tilde{V} + (j\omega C)\tilde{V}$$

$$\vdots$$

$$\tilde{V} = \frac{\tilde{I}_s}{\frac{1}{R} + \frac{1}{j\omega L} + j\omega C} = \frac{\tilde{I}_s}{\underbrace{\frac{1}{R}}_{\text{real}} + j\underbrace{\left(\omega C - \frac{1}{\omega L}\right)}_{\text{Imaginary}}}$$

$$\frac{1}{j\omega L} \Downarrow -\frac{j}{\omega L}$$

$\Rightarrow$ Convert to time domain:

$$i_s(t) = A\cos\left(\omega t + \frac{\pi}{3}\right)\ A$$

$$i_s(t) = \mathrm{Re}\{\tilde{I}_s e^{j\omega t}\}\ A$$

$$\tilde{I}_s = Ae^{j\pi/3}$$

$$\tilde{V} = \frac{\tilde{I}_s}{\underbrace{\left(\frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)\right)}_{}}$$

Magnitude: $\sqrt{\left(\frac{1}{R}\right)^2 + \left(\omega C - 1/\omega L\right)^2} = B$

Phase

$$\tan\theta = \frac{\text{imaginary}}{\text{real}} = \frac{\omega C - 1/\omega L}{1/R}$$

$$\tan\theta = R(\omega C - 1/\omega L)$$

$$\theta = \tan^{-1}\left(R(\omega C - 1/\omega L)\right)$$

$$v(t) = Re\left\{ \frac{I_o e^{j\pi/3} e^{j\omega t}}{B e^{j\theta}} \right\}$$

$$v(t) = Re\left\{ \frac{I_o}{B} e^{j\omega t} e^{j(\pi/3 - \theta)} \right\}$$

$$v(t) = \frac{I_o}{\sqrt{(1/R)^2 + (\omega C - 1/\omega L)^2}} \cos\left(\omega t + \frac{\pi}{3} - \theta\right)$$

$$\theta = \tan^{-1}\left(R(\omega C - 1/\omega L)\right)$$

Chracteristic Impedance:

⇒ Example - Reflection of series RC Load

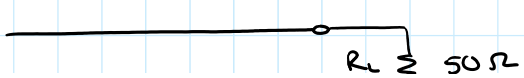
Consider a $100\,\Omega$ transmission line, $Z_o = 100\,\Omega$

Connected to load: resistor $50\,\Omega$ in series with $10\,pF$ capacitor:

$\rightarrow R_L = 50\,\Omega$

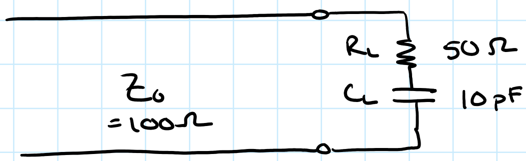
$C_L = 10\,pF = 10^{-11}\,F$

$f = 100\,MHz = 10^8\,Hz$



Q: Find the reflection

$f = 100$ MHz $= 10^8$ Hz



Q: Find the reflection coefficient for 100 MHz signal.

Knowns: Z_0, R_L, C_L, f

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{Z_L/Z_0 - 1}{Z_L/Z_0 + 1} = \frac{z_L - 1}{z_L + 1}, \quad z_L = \frac{Z_L}{Z_0}$$

$$z_L = \frac{Z_L}{Z_0}, \quad Z_L = R_L - j/\omega C_L$$

$$z_L = \frac{R_L - j/\omega C_L}{Z_0} = \frac{1}{100}\left(50 - j\frac{1}{2\pi \times \underbrace{10^8}_{f} \times \underbrace{10^{-11}}_{C_L}}\right)$$

$$z_L = 0.5 - j1.59 \ \Omega$$

Solving reflection

$$\Gamma = \frac{z_L - 1}{z_L + 1} = \frac{(0.5 - j1.59) - 1}{(0.5 - j1.59) + 1} = \frac{-0.5 - j1.59}{1.5 - j1.59}$$

$$\Gamma = \frac{-0.5 - j1.59}{1.5 - j1.59} \longrightarrow A = \sqrt{(0.5)^2 + (1.59)^2} \Rightarrow -A$$

$$= 1.67$$

$$\phi_A = \tan^{-1}\left(\frac{1.59}{0.5}\right)$$

$$= 72.5^\circ$$

$$-Ae^{j\phi_A}$$

$$B = \sqrt{(1.5)^2 + (1.59)^2}$$

$$= 2.19$$

$$\phi_B = \tan^{-1}\left(\frac{-1.59}{\ }\right) = -46.7^\circ$$

$-Ae^{j\ldots}$

$$\phi_B = \tan^{-1}\left(\frac{-1.59}{1.5}\right) = -46.7^\circ$$

$Be^{j\phi_B}$

$$\Gamma = \frac{-1.67e^{j72.6^\circ}}{2.19e^{-j46.7^\circ}} = -0.76e^{j119.3^\circ}$$

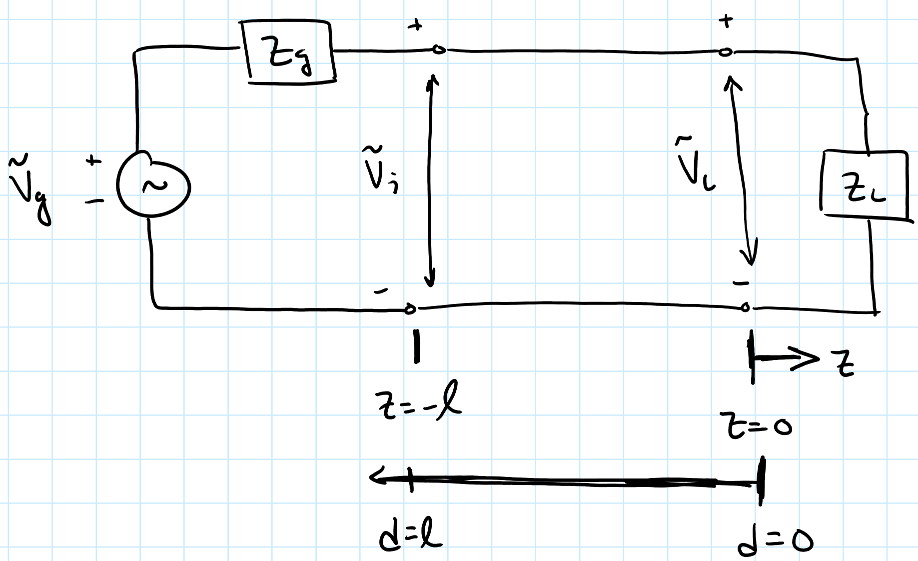
$$\Gamma = e^{-j180^\circ}e^{j119.3^\circ}(0.76)$$

$$\boxed{\Gamma = 0.76e^{-j60.7} = 0.76\angle -60.7^\circ}$$

$$\frac{V_0^-}{V_0^+} = \Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

Standing Waves:

$$Z_0 = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}}$$



★ $$|\tilde{V}(d)| = |V_0^+|\left[1 + |\Gamma|^2 + 2|\Gamma|\cos(2\beta d - \theta_r)\right]^{1/2}$$

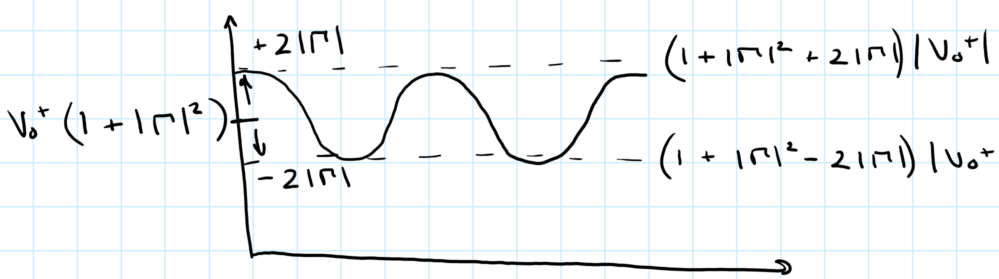
★ $$|\tilde{V}(d)| = |V_0^+|\left[1 + |\Gamma|^2 + 2|\Gamma|\cos(2\beta d - \theta_r)\right]^{1/2}$$

★ $$|\tilde{I}(d)| = \frac{|V_0^+|}{Z_0}\left[1 + |\Gamma|^2 - 2|\Gamma|\cos(2\beta d - \theta_r)\right]^{1/2}$$

→ Maximum: $|\tilde{V}(d)|$ max occurs when: $\underline{2\beta d - \theta_r = 2n\pi}$

→ Minimum $|\tilde{V}(d)|$ min occurs when: $2\beta d - \theta_r = (2n+1)\pi$

$$\Big[\underbrace{1 + |\Gamma|^2}_{\text{DC offset}} + \underbrace{2|\Gamma|\cos(2\beta d - \theta_r)}_{\text{oscillation}}\Big]^{1/2}$$



Note: spatial (λ) of a standing wave $= \lambda/2$

$$|\tilde{V}|_{max} = |V_0^+|(1 + |\Gamma|)$$

$\Rightarrow 2\beta d_{max} - \theta_r = 2\pi n$

$$\left[d_{max} = \frac{\theta_r + 2n\pi}{2\beta} = \frac{\theta_r\lambda}{4\pi} + \frac{n\lambda}{2}\right], \quad \beta = \frac{2\pi}{\lambda}$$

$$\begin{cases} n = 1, 2, \ldots & \text{if } \theta_r < 0 \\ n = 0, 1, 2 \ldots & \text{if } \theta_r \geq 0 \end{cases} \qquad -\pi \leq \theta_r \leq \pi$$

$$\theta_r \geq 0: \quad \boxed{d_{max} = \frac{\theta_r \lambda}{4\pi} \quad \Rightarrow n = 0}$$

$$\theta_r < 0: \quad \boxed{d_{max} = \frac{\theta_r \lambda}{4\pi} + \frac{\lambda}{2} \quad \Big\}\ \text{this occurs at } n=1 \text{ first}}$$

$$|\tilde{V}_{min}| = |V_0^+|(1 - |\Gamma|)$$

$$\boxed{d_{min} \begin{cases} d_{max} + \frac{\lambda}{4} & \text{if } d_{max} < \lambda/4 \\ d_{max} - \frac{\lambda}{4} & \text{if } d_{max} \geq \lambda/4 \end{cases}}$$